Contribution of hydrodynamic modes to the thermal properties of the neutron star crust

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MODE - SNR - PWN Workshop, November 16^{th}

1. Specific heat of a neutron star
   - Introduction
   - The inner crust
   - Thermal properties of the pasta phase
2. Hydrodynamic modes in "lasagna"
   - Characteristics of "lasagna"
   - Superfluid hydrodynamics
   - Solving the equations
   - Excitation spectrum
3. Specific heat
4. Conclusion

One of the neutron star observables is the surface temperature which can give constraints on the thermal evolution estimating its age.

- Specific heat is one of the elements to study the thermal evolution of neutron star
- Specific heat is a sum over different contributions from the different excitations (nuclei, phonons, electrons,...)
- Shortly after the birth the core contains still a lot of energy which escapes through the crust $\Longrightarrow$ I will study thermal properties of the crust.

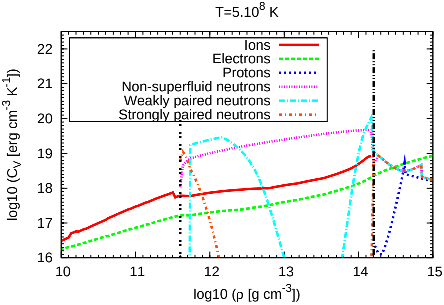

Figure: Specific heat contribution as a function of the density at $T = 5.10^8$K, thanks to Morgane Fortin

We will be interested in the inner crust which contains the structure called "pasta phase".

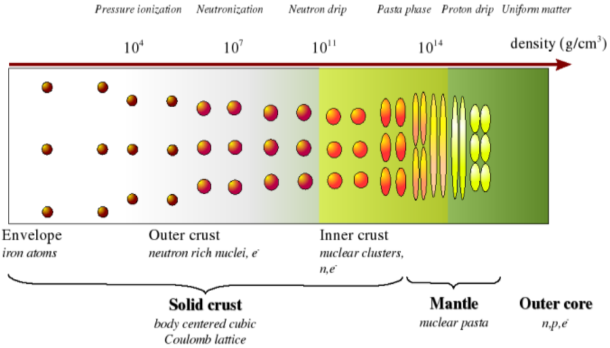

Figure: Neutron star crust

- This part of the crust is characterised by the transition from homogeneous matter to the lattice of atomic nuclei.
- Pasta phase = very deformed nuclei.

What are the different contributions to thermal properties?

- Paired nucleons: contribution strongly suppressed due to pairing gap.
- Contribution of ions, electrons and free neutrons to specific heat
- But superfluidity $\Longrightarrow$ low energy collective excitations called hydrodynamic modes.
- These modes are first order perturbations in density and propagate at sound velocity.

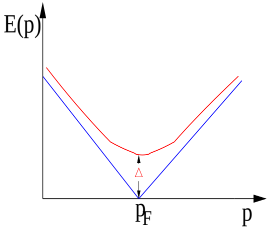

Figure: Energy gap of pairing.

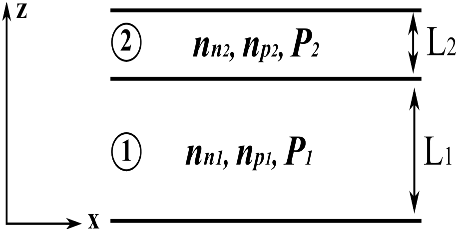

Figure: Representation of "lasagna"

I take the condition of:

- Lasagna: periodic alternance of two slabs("gazeous" and "liquid") with different proton and neutron densities $\Longrightarrow$ different thermodynamical properties
- Zero temperature approximation $\Longrightarrow$ neutrons and protons are treated as superfluids.
- Superfluid hydrodynamics approximation in each slab
- Non-relativistic approximation.

Two basic equations for deriving superfluid hydrodynamics:

- Conservation of particle number: $\partial_\mu \ n^\mu = 0$
- Energy-momentum conservation (Euler equation): $\partial_\mu T^{\mu\nu} = 0$ with $T^\mu{}_\nu = P\delta^\mu{}_\nu + \sum_{x=n,p} n^\mu_x \mu^x_\nu$

Characteristics of hydrodynamics with two superfluid components (n,p):

- No viscosity.
- Entrainment between the two fluids: non dissipative interaction which misalign velocities and momentum.

Parameters appearing in the hydrodynamic equations are calculated within a Landau-Fermi liquid model. Relativistic Mean Field interaction with $\sigma - \omega - \rho - \delta$ mesons is employed with density dependent parameters defined in Avancini et al. (2009).

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At $T \sim 10^8$K $\Longrightarrow$ time period of modes $\ll$ characterisic time of β-interaction, relaxation time
$\Longrightarrow$ Fluids are inviscid
$\Longrightarrow$ Contact is maintained
$\Longrightarrow$ Continuity of perpendicular fluid velocities and continuity of chemical potentials

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3. Due to invariance of the systeme along x axis we use the Snell-Descartes relation for refraction/reflection waves.
4. We use the Floquet-Bloch theorem to take into account the periodicity ($U(\overrightarrow{r} + \overrightarrow{L}) = U(\overrightarrow{r})e^{i\overrightarrow{q}\cdot\overrightarrow{L}}$ where $\overrightarrow{L}$ is the periodicity).

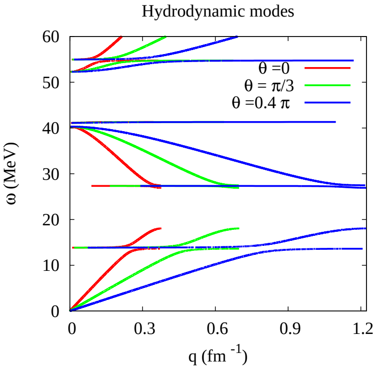
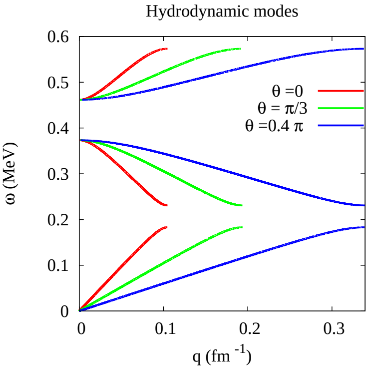


Figure: Baryonic density $n_b = 0.0804fm^{-3} \sim \frac{\rho_0}{2}$.



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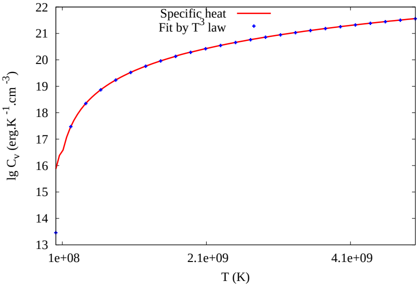
Taking Bose distribution for this hydrodynamics modes we can integrate over all momenta in order to obtain the specific heat contribution.



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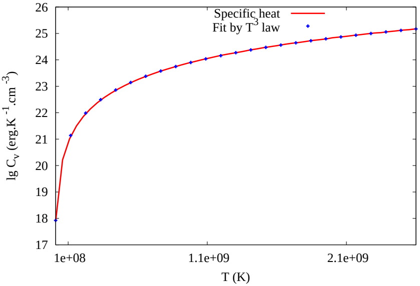

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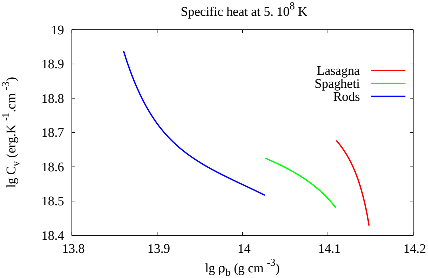
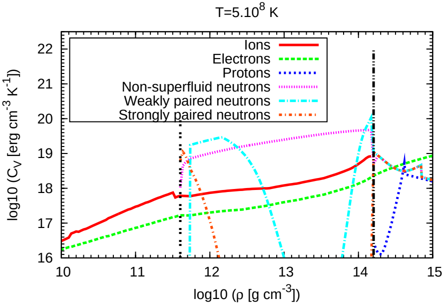


Figure: Specific heat contribution as a function of the density at $T = 5.10^8$K



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I have introduced a formalism for wave propagation in "lasagna" taking into account superfluidity and the periodic structure. The dispersion relations show interesting acoustic and optic branches. The calculation of the specific heat, which follow a classical temperature dependance for bose distribution,give a non negligible contribution.
Perspective:

- Resolve the problem for other geometrical structures.
- Estimate the effect of this specific heat to the thermal evolution of the neutron star
- Non-zero temperature $\Longrightarrow$ addition of a "normal fluid".