Thermalization time and specific heat of neutron stars crust

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`arXiv/nucl-th/0910.5488`

MODE-SNR-PWN 2010

Context (1)

Study of the cooling of a neutron star (NS) with fast cooling. Cooling time essentially determined by the properties of the inner-crust ie. :

- the thickness,
- the properties of the baryonic matter.

Composition of the inner-crust :

- ultrarelativistic electrons,
- unbound neutrons that can be superfluid,
- nuclear clusters, whose influence on the superfluid properties has to be taken into account.

Context (2)

- Solve the relativistic heat equation in the whole NS using NSCool[1] (D. Page),
- with a model of NS that is almost completely consistent (SLy4 nuclear interaction),
- using new calculations for the specific heat of unbound neutrons in the inner-crust.
- → estimation of the cooling time.

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[1] available on `http://www.astroscu.unam.mx/neutrones/NSCool/`

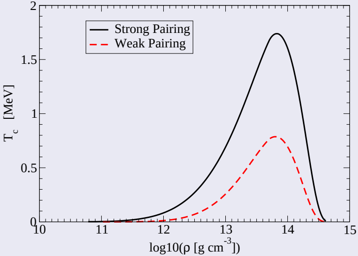
$^1S_0$ neutron pairing

HFB-FT calculations

- Mean field : Skyrme force SLy4 (Chabanat et al. 1997),
- Nuclear clusters : WS cells from Negele & Vautherin (1973),
- Pairing correlations :

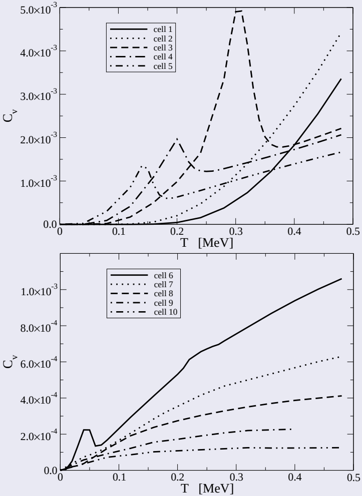
$$V(\mathbf{r}-\mathbf{r}') = V_0\left[1-\eta\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right]\delta(\mathbf{r}-\mathbf{r}'),$$

with V_0, η and α simulating two pairing scenarios :

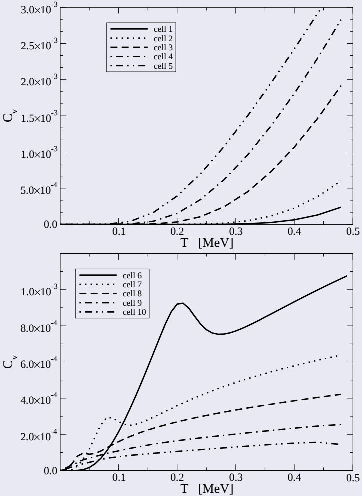


$^1S_0$ neutron pairing

Weak pairing



Strong pairing



Neutron star model

Equation of state (EoS) :

- Core : Douchin & Haensel (2001)
  - based on the SLy4 effective nuclear interaction (the same as in the C_V calculations),
  - npeμ composition.
- Inner-crust : Negele & Vautherin (1973)
  - $4 \times 10^{11} \leq \rho \leq\ 1.6 \times 10^{14}$ g cm^{-3}
  - density functional,
  - Hartree-Fock calculations.
- Outer-crust : Haensel, Zdunik & Dobaczewski (1989)
  - Skyrme effective nucleon-nucleon interaction (Dobaczewski, Flocard & Treiner, 1984),
  - Hartree-Fock-Bogoliubov (HFB) calculations.
- Effective mass : Skyrme nuclear interaction.

Cooling model

Heat equation (Thorne, 1977)

$$\frac{\partial}{\partial r}\left(\frac{Kr^2}{\Gamma(r)}e^{\phi}\frac{\partial}{\partial r}\left(Te^{\phi}\right)\right)=r^2\Gamma(r)e^{\phi}\left(C_V\frac{\partial T}{\partial t}+e^{\phi}Q_\nu\right),$$

- $\Gamma=\left(1-2Gm(r)/rc^2\right)^{-1/2}$, ϕ the gravitational potential,
- K the thermal conductivity,
- Q_ν the neutrino emissivity,
- C_V the specific heat.

Boundary conditions :

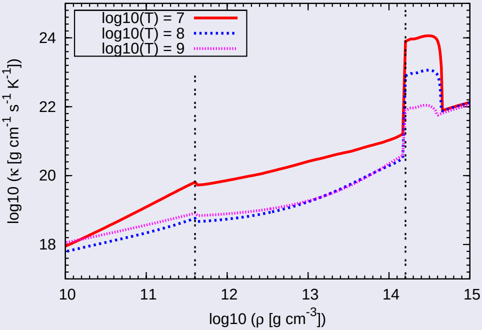
- $T(r,t=0)=T_i$
- $\rho=10^{10}$ g cm^{-3}, model of non-accreted envelope (Potekhin et al. 1997).

Cooling model

Thermal conductivity

Core :

- electrons & muons (Shternin & Yakovlev, 2007)
- nucleons (Baiko et al. 2001)



Crust :

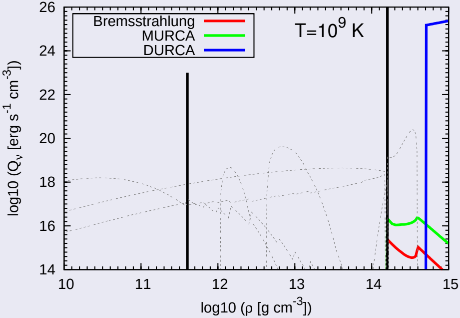
- electron-ion (Gnedin et al. 2001)
- electron-electron (Shternin & Yakovlev, 2006)

Cooling model

Neutrino emissivity (1)

Core :

- bremsstrahlung processes,
- MURCA,
- DURCA imposed for $\rho \geq 5 \times 10^{14}$ g cm^{-3} → **fast cooling**.

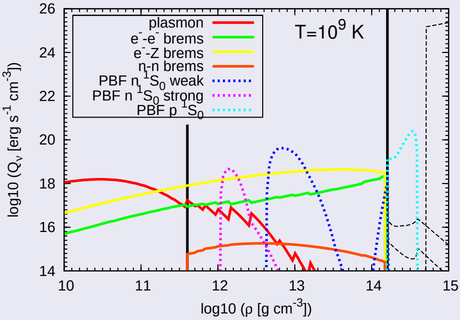


Cooling model

Neutrino emissivity (2)

Crust :

- plasmon decay,
- e^--e^-, e^--Z & n-n bremsstrahlung.



Superfluidity :

- reduction of the emissivities,
- Cooper pair breaking and formation processes (PBF).

Cooling model

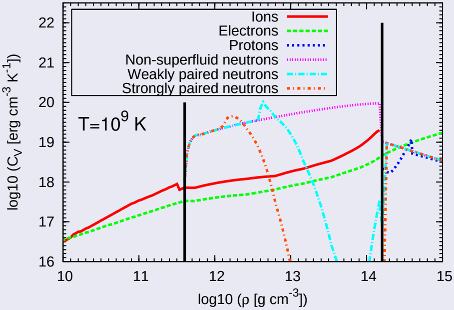
Specific heat (1)

Electrons :

- C_V of a uniform, degenerate gas.

Ions in the crust :

- solid-liquid phase transition included,



Protons in the core :

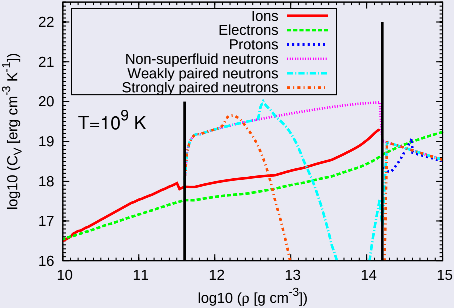
- $^1S_0$ pairing from Takatsuka (1973),

Cooling model

Specific heat (2)

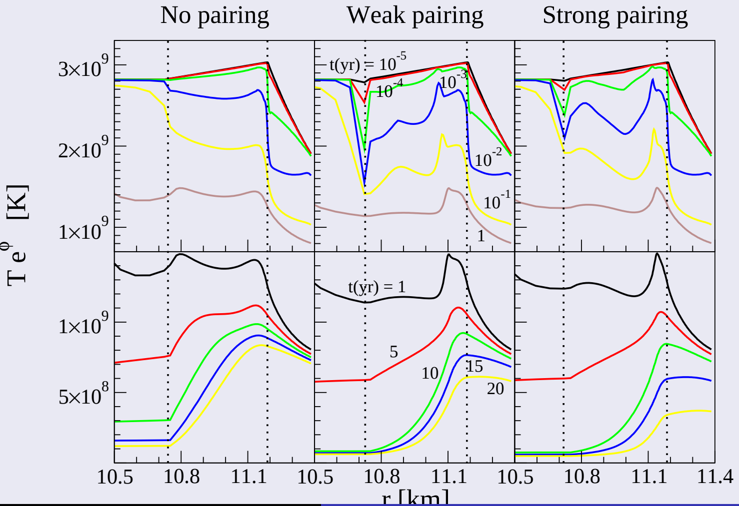
Unbound neutrons :

- in the core : $^3P_2$ pairing : model "a" from Page et al. 2004 with $T_c^{\max} \sim 10^9$ K,
- in the inner-crust : $^1S_0$ pairing : fits of the previous calculations.



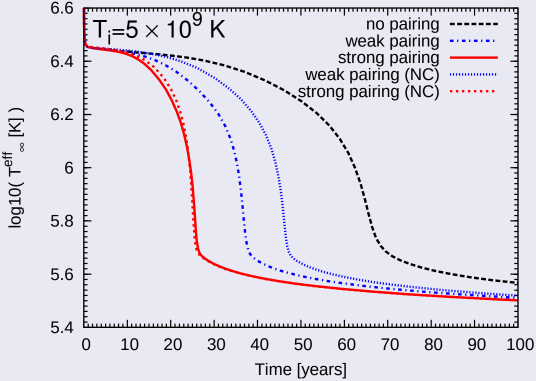
Crust thermalization

M=1.6 $M_\odot$ & $T_i = 5 \times 10^9$ K



Crust thermalization

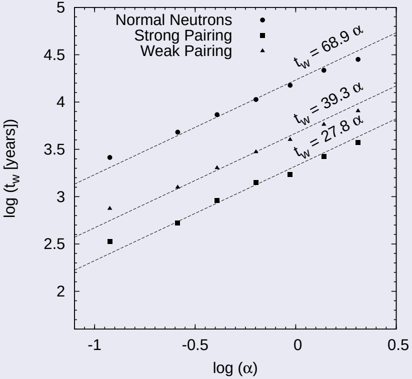
Cooling curves & pairing scenarios - M=1.6 $M_\odot$



Cooling time t_w : $T^{\rm eff}_\infty(t=t_w)$ has its most negative slope.

Scaling relations

$\mathrm{M} \in [1.4, 2.0]\ \mathrm{M}_\odot$ & $\mathrm{T_i} = 5 \times 10^9$ K



Lattimer et al. 1994, Gnedin et al. 2001

Scaling parameter : $\alpha = \left(\frac{\Delta R_{\mathrm{crust}}}{1\ \mathrm{km}}\right)^2 \left(1 - \frac{2GM}{c^2 R}\right)^{-3/2}$

Conclusion (1)

New calculations of the specific heat of neutrons in the crust :

- HFB at finite temperature;
- inclusion of the effects of :
  - the temperature,
  - the nuclear clusters,
  - the pairing correlations.

Study the thermalization of NS crusts
in the fast cooling scenario
for an almost completely consistent model (SLy4).

Conclusion (2)

Results

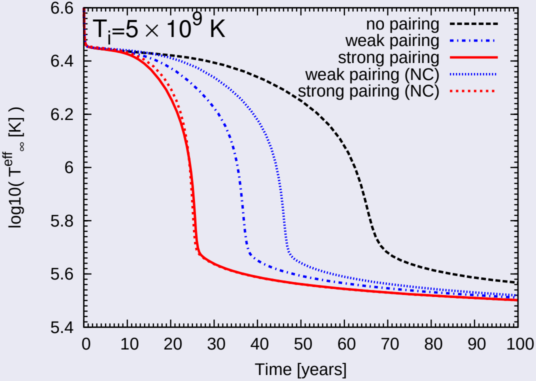
- The pairing correlations have a strong influence on cooling.
- The cluster structure of the inner-crust has a non-trivial influence.

Perspective

- Performing cooling calculations in WS cells calculated for the SLy4 force.

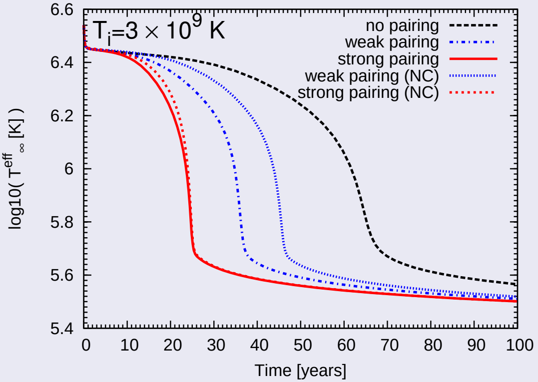
Crust thermalization

Cooling curves & pairing scenarios - M=1.6 $M_\odot$



Crust thermalization

Cooling curves & pairing scenarios - M=1.6 $M_\odot$



$^1S_0$ neutron pairing

Parametrization of C_V^n

$$C_V^n = (1 - x_{\rm cl})C_V^{\rm cl} + x_{\rm cl} R C_V^q$$

with :

- $C_V^{\rm cl}$ the specific of non-superfluid unbound neutrons in the classical regime,
- C_V^q the specific of non-superfluid unbound neutrons in the quantum regime,
- $x_{\rm cl}$ the factor describing the transition between classic and quantum behavior,
- R the factor simulating the reduction due to pairing correlations.

$^1S_0$ neutron pairing

Parametrization of C_V^n

$$C_V^n = x_{\mathrm{cl}} R C_V^q + (1 - x_{\mathrm{cl}}) C_V^{\mathrm{cl}}$$

with :

- the factor describing the transition between classic and quantum behavior,

$$x_{cl} = \left(1 + e^{5\left(\frac{\pi T}{\varepsilon_F} - 1\right)}\right)^{-1}$$

- with $\varepsilon_F = \hbar^2 k_F^2 / 2m_n^*$ the Fermi energy at zero T.
- For normal, unbound neutrons :

$$C_V^q = \frac{1}{6}\left(\frac{2m_n^*}{\hbar^2}\right)^{3/2} \varepsilon_F^{1/2} T \times \left[1 - \frac{7}{40}\left(\frac{\pi T}{\varepsilon_F}\right)^2 - \frac{155}{896}\left(\frac{\pi T}{\varepsilon_F}\right)^4\right].$$

$^1S_0$ neutron pairing

Parametrization of C_V^n

$$C_V^n = x_{\rm cl} R C_V^q + (1 - x_{\rm cl}) C_V^{\rm cl}$$

with :

- For classic neutrons :

$$C_V^{\rm cl} = \frac{3}{2}\rho_{\rm gas},$$

  - with, for $T < T_{gas} = 5.5$ MeV,
$$\rho_{\rm gas} = \rho_n(T=0) + \frac{T}{T_{\rm gas}}\left(\rho_{\rm max} - \rho_n(T=0)\right),$$
  - for $T > T_{gas} = 5.5$ MeV,
$$\rho_{\rm gas} = \rho_{\rm max}.$$
  - with $\rho_{\rm max}$ for neutrons uniformly distributed in the cell.

$^1S_0$ neutron pairing

Parametrization of C_V^n

$$C_V^n = x_{\rm cl} R C_V^q + (1 - x_{\rm cl}) C_V^{\rm cl}$$

with the factor simulating the reduction due to pairing correlations :

$$R = R_{\rm YL}(u) f_1(T, \Delta_o, a_0, a_1, a_3)\,(1 - f_2(T, \Delta_o, a_0, a_2, a_3))\,,$$

where

- $R_{\rm YL}(u)$ is the superfluid reduction factor for uniform neutron matter (Levenfish et al., 1994),
- f_1 & f_2 are two functions describing the normal/superfluid transition, depending on :
  - Δ_o the pairing energy gap in the neutron gas at T=0,
  - a_0, a_1, a_2, a_3 four parameters fitted to reproduce the results the HFB calculations.

Scaling relations

$$t_w = \alpha t_1 \text{ with } \alpha = \left(\frac{\Delta R_{\rm crust}}{1 \text{ km}}\right)^2 \left(1 - \frac{2GM}{c^2 R}\right)^{-3/2}$$

$M = 1.5M_\odot$ & $T_i = 5 \times 10^9$ K

Model of neutron superfluidity	t_w	t_1
No superfluidity	76.3	66.4
Weak pairing	43.1	37.4
Strong pairing	24.7	21.5