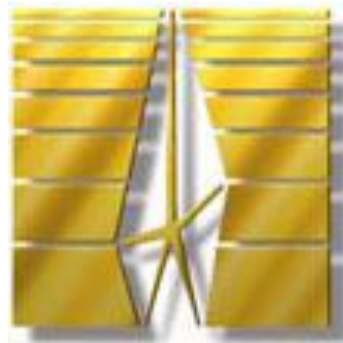


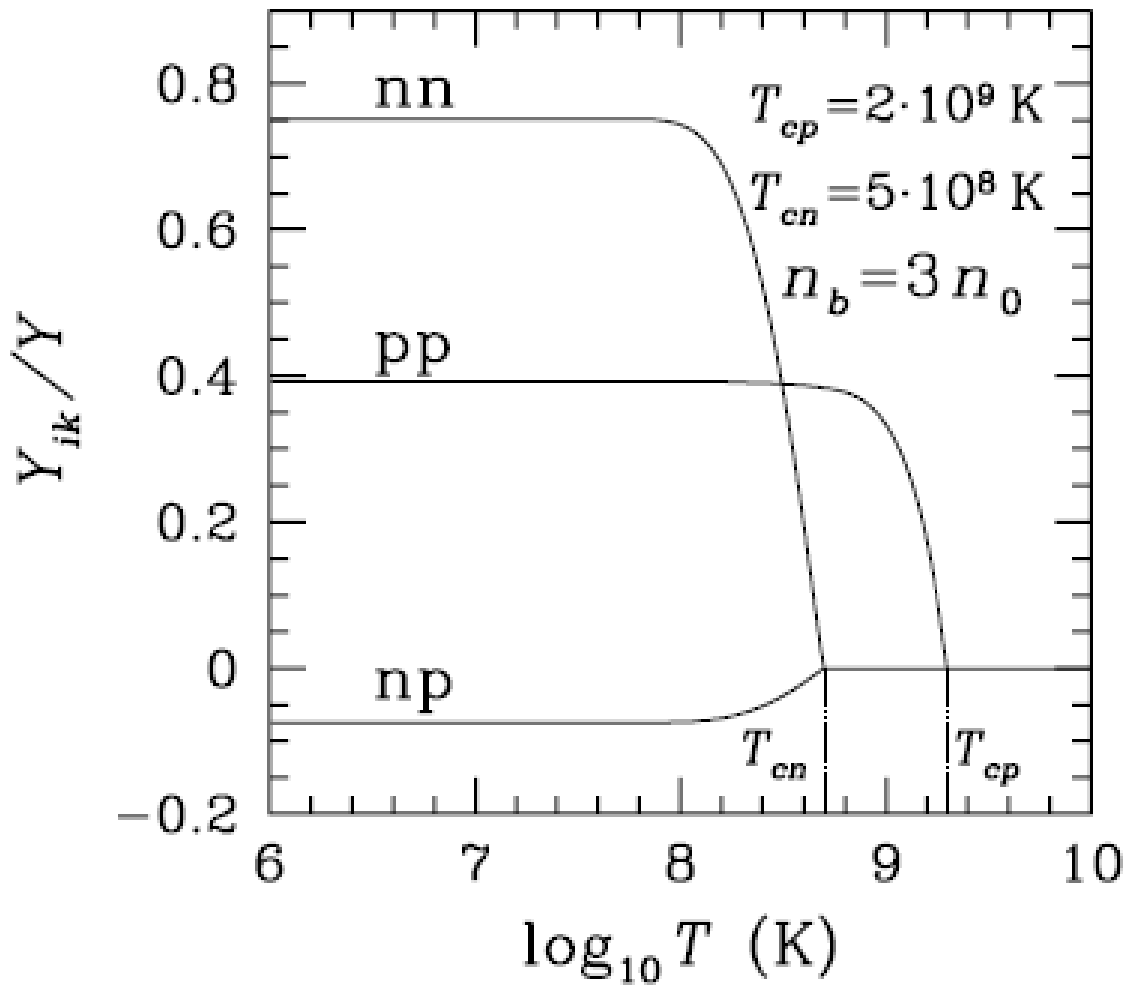
# Oscillations of superfluid neutron stars

**A.I. Chugunov, M.E. Gusakov**  
Ioffe Institute



**MODE-SNR-PWN Workshop, November 15-17, 2010**  
**Bordeaux, France**

# Superfluidity at $T \neq 0$



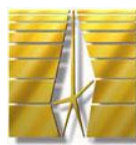
$$j_{(i)}^\mu = n_i u^\mu + Y_{ik} w_{(k)}^\mu$$

Strong temperature dependence

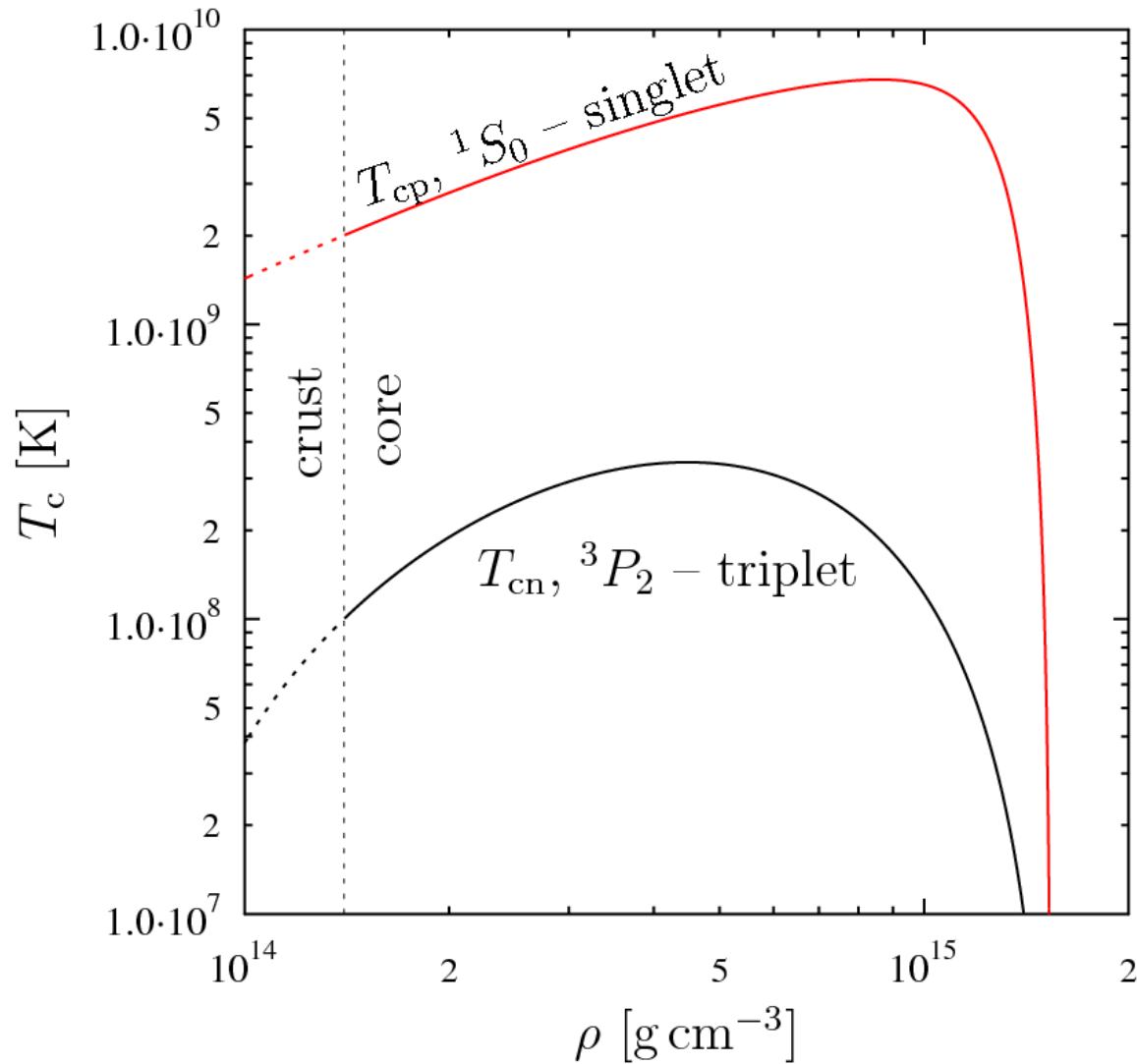


Temperature dependent pulsations!

From Gusakov, Kantor, Haensel (2009), PRC 80, 015803



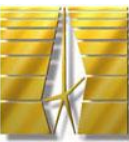
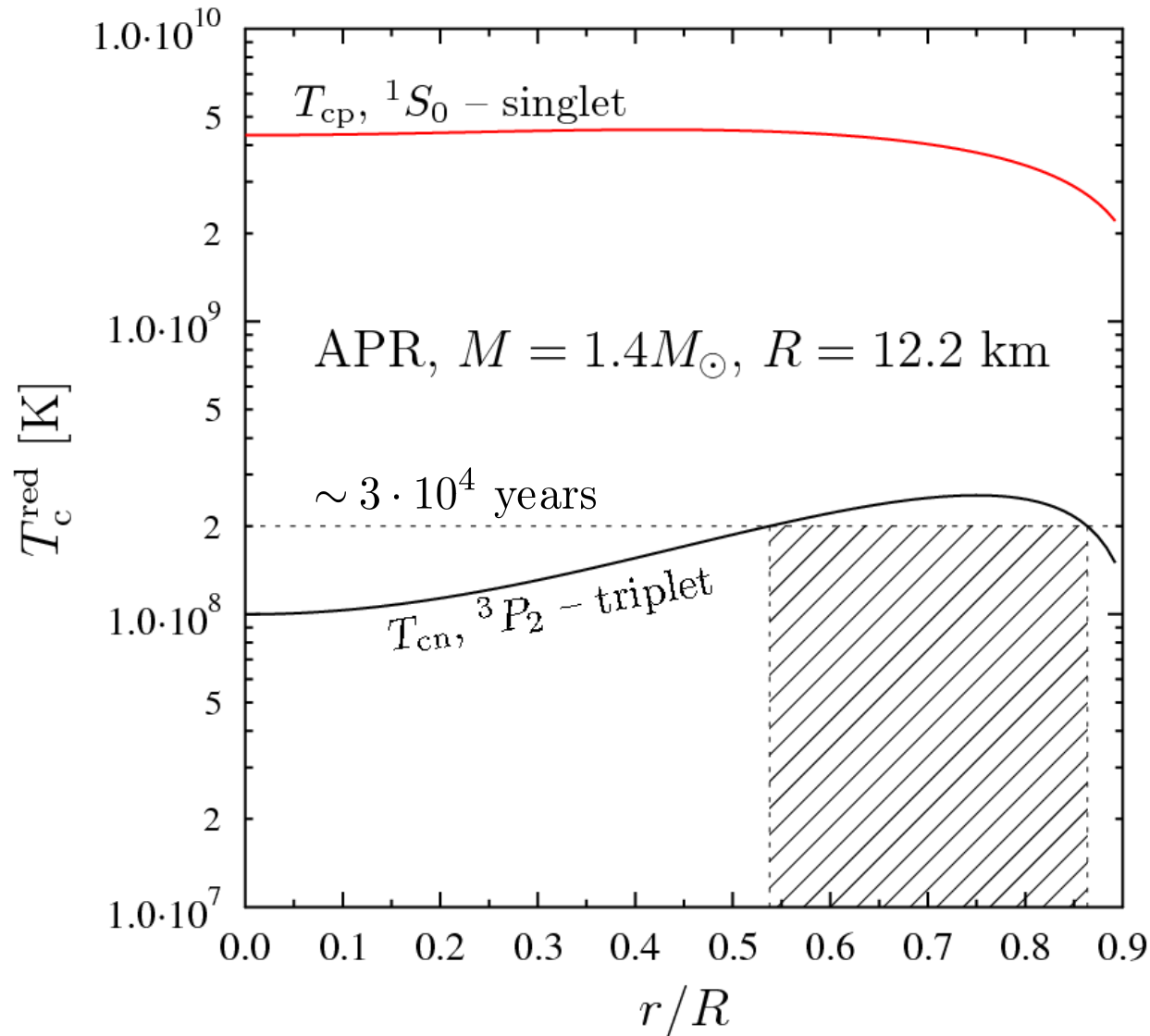
# Superfluidity in NS cores



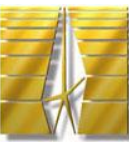
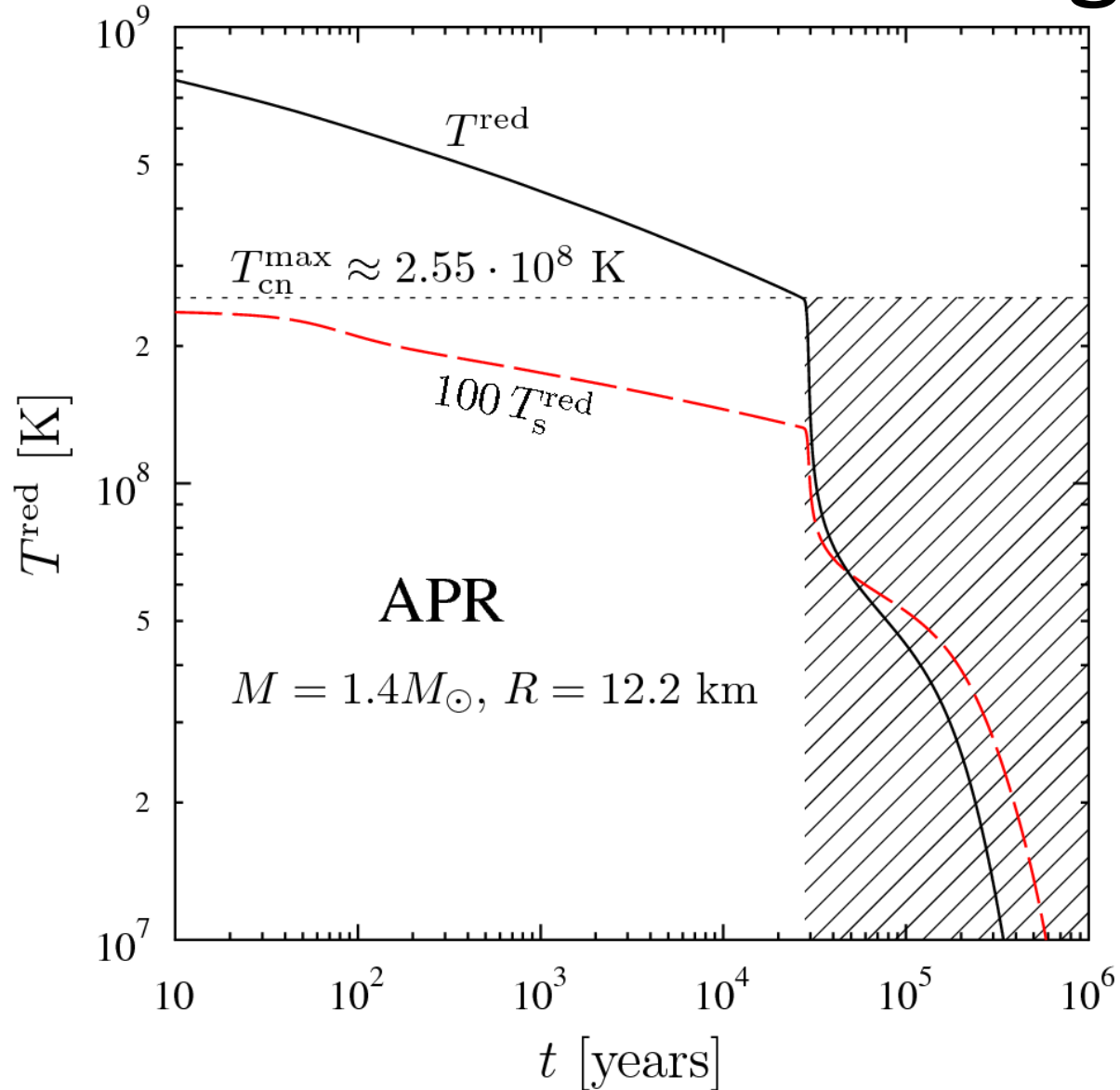
The model by Kaminker, Yakovlev, Gnedin (2002), A&A **383**, 1076



# Critical temperature vs radius



# Neutron star cooling



# Decoupling of superfluid and normal modes

Talk by M.E. Gusakov, E.M. Kantor arXiv:1007.2752

$$s \equiv \frac{n_{e0}}{n_{b0}} \frac{\partial P / \partial n_{e0}}{\partial P / \partial n_{b0}} \ll 1$$

$$j_{(b)}^{\mu} = n_b u^{\mu} + Y_{nk} w_{(k)}^{\mu}$$

**Normal**

$$\delta j_{(b)}^{\mu} \sim n_b \delta u^{\mu}$$

$$\frac{Y_{nk} w_{(k)}^{\mu}}{n_b \delta u^{\mu}} \sim 1$$

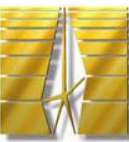
Frequencies and  $\delta j_{(b)}^{\mu}$  are the same as for normal star

**Modes**

**Superfluid**

$$\delta j_{(b)}^{\mu} \ll n_b u^{\mu}$$

$$\delta j_n^{\mu} \neq 0, \delta u^{\mu} \neq 0$$



# Equation for superfluid modes

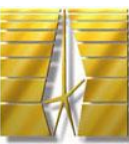
$$\delta\mu'' + \left( \frac{h'}{h} - \lambda' + \frac{2}{r} \right) \delta\mu' - e^\lambda \left[ \frac{l(l+1)}{r^2} + e^{-\nu/2} \frac{\omega^2}{h} \left( \frac{\partial\delta\mu}{\partial n_e} \right)^{-1} \right] \delta\mu = 0$$

$$\delta\mu = \mu_n - \mu_p - \mu_e$$

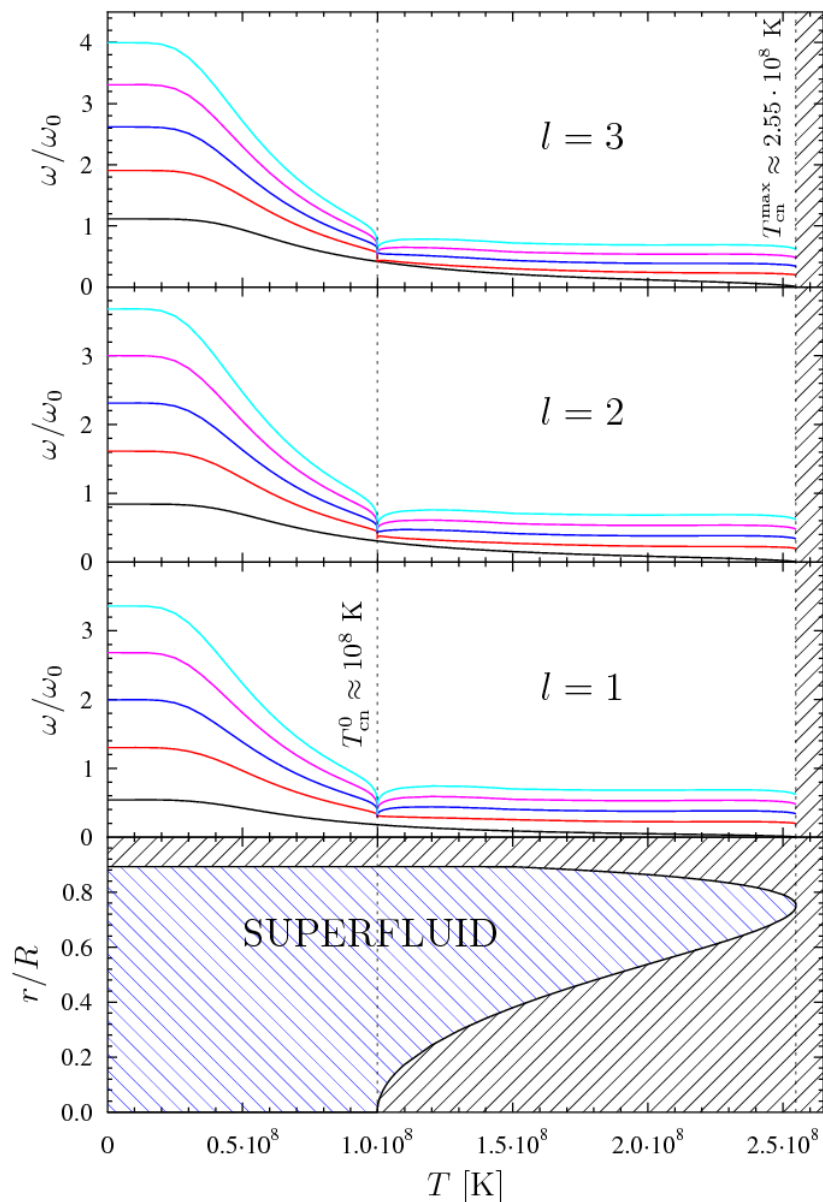
Microphysics

$$h = e^{\nu/2} \frac{n_e^2}{y n_b \mu_n}$$

$$y = \frac{n_b Y_{pp}}{\mu_n (Y_{nn} Y_{pp} - Y_{np} Y_{pn})} - 1$$

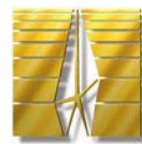


# Oscillation spectrum vs $T$



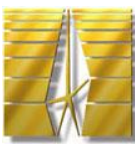
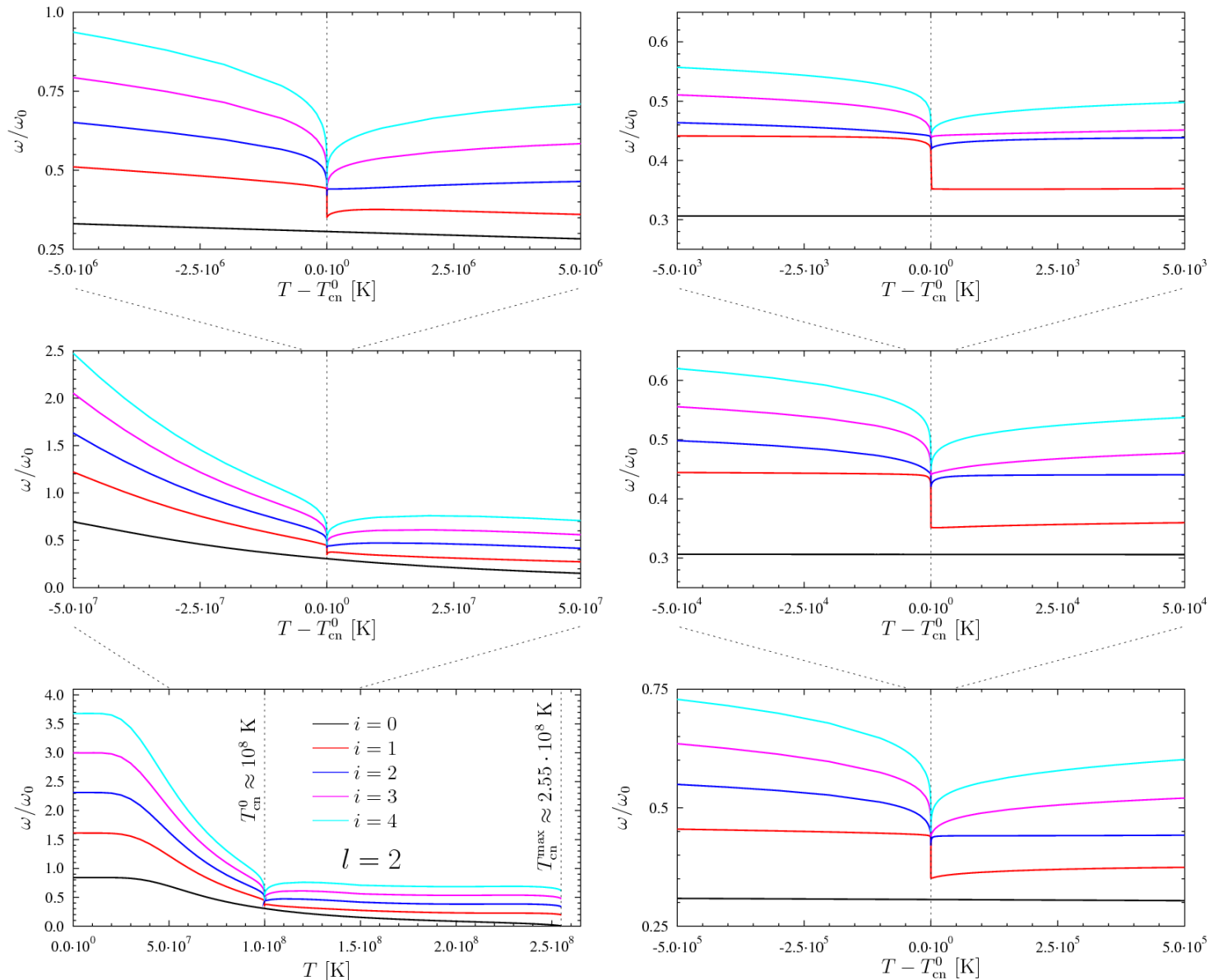
$$\omega_0 = c/R$$

- Frequencies are very sensitive to the variation of temperature
- Frequencies depend not only on the superfluid region, but also on the temperature at this region

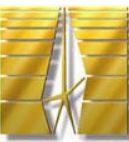
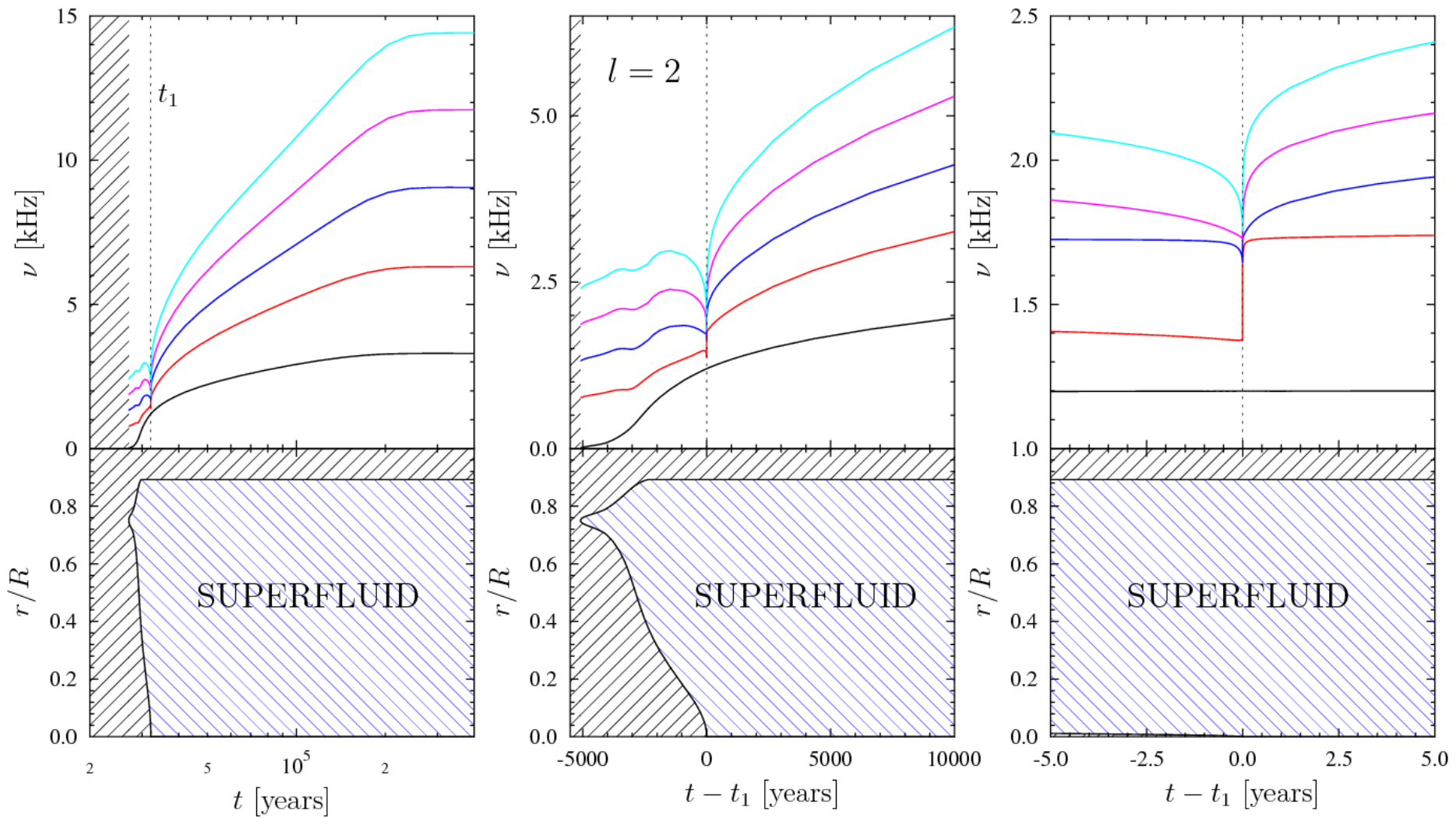




# Oscillation spectrum vs $T$



# Evolution of spectrum



# Damping of superfluid modes

Zero order in  $s$

$\delta u^\mu \neq 0$   $\longrightarrow$  Shear viscosity damping

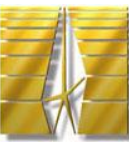
$\delta \mu \neq 0$   $\longrightarrow$  Bulk viscosity damping

$\delta g^{\mu\nu} = 0$   $\longrightarrow$  No gravitational waves

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Next order perturbations

$\delta g^{\mu\nu} \propto s$   $\longrightarrow$  Coupling with gravitational waves is strongly ( $\sim s^2 \sim 10^{-3}$ ) suppressed



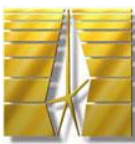
# Future perspectives of the decoupling approach

## *Normal modes*

- Frequencies and  $j_{(b)}^{\mu}$  can be calculated using the code designed to calculate pulsations of normal stars.
- The normal velocity  $u^{\mu}$  and  $w_{(k)}^{\mu}$  can be easily extracted from  $j_{(b)}^{\mu}$  with help of superfluid equation.
- The damping rates can be calculated, since  $u^{\mu}$  and  $w_{(k)}^{\mu}$  are known

## *Superfluid modes*

- Calculation of the gravitational waves from superfluid modes is a simple task, if one has a code calculating pulsations of a normal star
- Next order perturbation theory can be easily developed



# Results and conclusions

*Oscillations of redrealistic superfluid neutron stars are studied. We dont simplify microphysics and apply:*

- **realistic equation of state (APR)**
- **realistic density dependent models of critical temperatures**

*We demonstrate that:*

- The superfluid pulsations strongly depend on temperature
- The temperature dependence caused not only by variation of superfluid region, but also by the properties of the matter
- Temporal evolution of superfluid spectrum is fast
- The superfluid modes are dumped by viscosity in zero order in  $s$
- The emission of gravitational waves is supressed by a factor of  $s^2 \lesssim 10^{-3}$

