Thermalization time and specific heat of neutron stars crust

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Study of the cooling of a neutron star (NS) with fast cooling. Cooling time essentially determined by the properties of the inner-crust ie.:

- the thickness,
- the properties of the baryonic matter.

Composition of the inner-crust:

- ultrarelativistic electrons,
- unbound neutrons that can be superfluid,
- nuclear clusters, whose influence on the superfluid properties has to be taken into account.
Solve the relativistic heat equation in the whole NS using NSCool\(^1\) (D. Page),

- with a model of NS that is almost completely consistent (SLy4 nuclear interaction),
- using new calculations for the specific heat of unbound neutrons in the inner-crust.
- → estimation of the cooling time.

\(^1\)available on http://www.astroscu.unam.mx/neutrones/NSCool/
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Introduction

$^1S_0$ neutron pairing

HFB-FT calculations

- Mean field: Skyrme force SLy4 (Chabanat et al. 1997),
- Nuclear clusters: WS cells from Negele & Vautherin (1973),
- Pairing correlations:

\[
V(r - r') = V_0 \left[ 1 - \eta \left( \frac{\rho(r)}{\rho_0} \right)^\alpha \right] \delta(r - r'),
\]

with $V_0$, $\eta$, and $\alpha$ simulating two pairing scenarios:
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Neutron star model

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Scaling relations

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$^1S_0$ neutron pairing

![Graphs showing thermalization time and specific heat for weak and strong pairing](image-url)
Neutron star model

Equation of state (EoS):

- **Core**: Douchin & Haensel (2001)
  - based on the SLy4 effective nuclear interaction (the same as in the $C_V$ calculations),
  - npe$\mu$ composition.

- **Inner-crust**: Negele & Vautherin (1973)
  - $4 \times 10^{11} \leq \rho \leq 1.6 \times 10^{14}$ g cm$^{-3}$
  - density functional,
  - Hartree-Fock calculations.

- **Outer-crust**: Haensel, Zdunik & Dobaczewski (1989)
  - Skyrme effective nucleon-nucleon interaction (Dobaczewski, Flocard & Treiner, 1984),
  - Hartree-Fock-Bogoliubov (HFB) calculations.

- **Effective mass**: Skyrme nuclear interaction.
Cooling model

Heat equation (Thorne, 1977)

\[
\frac{\partial}{\partial r} \left( \frac{K r^2}{\Gamma(r)} e^\phi \frac{\partial}{\partial r} (T e^\phi) \right) = r^2 \Gamma(r) e^\phi \left( C_V \frac{\partial T}{\partial t} + e^\phi Q_\nu \right),
\]

- \( \Gamma = (1 - 2Gm(r)/rc^2)^{-1/2} \), \( \phi \) the gravitational potential,
- \( K \) the thermal conductivity,
- \( Q_\nu \) the neutrino emissivity,
- \( C_V \) the specific heat.

Boundary conditions:
- \( T(r, t = 0) = T_i \)
- \( \rho = 10^{10} \text{ g cm}^{-3} \), model of non-accreted envelope (Potekhin et al. 1997).
Cooling model

Thermal conductivity

Core:
- electrons & muons (Shternin & Yakovlev, 2007)
- nucleons (Baiko et al. 2001)

Crust:
- electron-ion (Gnedin et al. 2001)
- electron-electron (Shternin & Yakovlev, 2006)
Neutrino emissivity (1)

Core :
- bremsstrahlung processes,
- MURCA,
- DURCA imposed for \( \rho \geq 5 \times 10^{14} \, \text{g cm}^{-3} \) → fast cooling.

Graph showing
- \( \log_{10} (Q_{\nu} \, \text{[erg s}^{-1} \text{cm}^{-3}]) \)
- \( \log_{10} (\rho \, \text{[g cm}^{-3}]) \)
- \( T=10^9 \, \text{K} \)
- Bremsstrahlung, MURCA, DURCA

\( \rho \geq 5 \times 10^{14} \, \text{g cm}^{-3} \)
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Neutrino emissivity (2)

Crust:
- plasmon decay,
- $e^- - e^-$, $e^- - Z$ & $n-n$ bremsstrahlung.

Superfluidity:
- reduction of the emissivities,
- Cooper pair breaking and formation processes (PBF).

Cooling model

$T = 10^9$ K

Plotted curves:
- plasmon
- $e^- - e^-$ brems
- $e^- - Z$ brems
- $n-n$ brems
- PBF $n^1S_0$ weak
- PBF $n^1S_0$ strong
- PBF $p^1S_0$
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Specific heat (1)

Electrons:

- $C_V$ of a uniform, degenerate gas.

Ions in the crust:

- solid-liquid phase transition included,

Protons in the core:

- $^1S_0$ pairing from Takatsuka (1973),
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Specific heat (2)

Unbound neutrons:

- in the core:
  $^3P_2$ pairing: model "a" from Page et al. 2004 with $T_c^{\text{max}} \sim 10^9$ K,

- in the inner-crust:
  $^1S_0$ pairing: fits of the previous calculations.

Unbound neutrons:

- in the core:
  $^3P_2$ pairing: model "a" from Page et al. 2004 with $T_c^{\text{max}} \sim 10^9$ K,

- in the inner-crust:
  $^1S_0$ pairing: fits of the previous calculations.
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\[ M=1.6 \, M_\odot \, \text{&} \, T_i = 5 \times 10^9 \, K \]

No pairing    Weak pairing    Strong pairing

\begin{tabular}{l}
\textbf{\textit{T}} \wedge \textbf{\textit{e}} \\
- \textbf{\textit{φ}} [K] \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{r}} [km] \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 10^{-5} \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 10^{-4} \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 10^{-3} \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 10^{-2} \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 10^{-1} \\
\end{tabular}

\begin{tabular}{l}
\textbf{\textit{t}} (yr) = 1 \\
\end{tabular}

No pairing

Weak pairing

Strong pairing
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Cooling time $t_w$: $T_{\infty}(t = t_w)$ has its most negative slope.
Scaling relations

$M \in [1.4, 2.0]~M_\odot$ & $T_i = 5 \times 10^9$ K

Lattimer et al. 1994, Gnedin et al. 2001
Scaling parameter : $\alpha = \left( \frac{\Delta R_{\text{crust}}}{1 \text{ km}} \right)^2 \left( 1 - \frac{2GM}{c^2 R} \right)^{-3/2}$
Conclusion (1)

New calculations of the specific heat of neutrons in the crust:
- HFB at finite temperature;
- inclusion of the effects of:
  - the temperature,
  - the nuclear clusters,
  - the pairing correlations.

Study the thermalization of NS crusts in the fast cooling scenario for an almost completely consistent model (SLy4).
Conclusion (2)

Results
- The pairing correlations have a strong influence on cooling.
- The cluster structure of the inner-crust has a non-trivial influence.

Perspective
- Performing cooling calculations in WS cells calculated for the SLy4 force.
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Cooling curves & pairing scenarios - \( M = 1.6 \, M_\odot \)

\[ T_i = 5 \times 10^9 \, K \]

\[ \log_{10}(T_{\text{eff}}) \, [K] \]

Time [years]

\( T_i = 5 \times 10^9 \, K \)

- no pairing
- weak pairing
- strong pairing
- weak pairing (NC)
- strong pairing (NC)
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Parametrization of $C_V^n$

$$C_V^n = (1 - x_{cl})C_V^{cl} + x_{cl}RC_V^q$$

with:

- $C_V^{cl}$ the specific of non-superfluid unbound neutrons in the classical regime,
- $C_V^q$ the specific of non-superfluid unbound neutrons in the quantum regime,
- $x_{cl}$ the factor describing the transition between classic and quantum behavior,
- $R$ the factor simulating the reduction due to pairing correlations.
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$1S_0$ neutron pairing

Parametrization of $C_V^n$

\[ C_V^n = x_{cl} R C_V^q + (1 - x_{cl}) C_V^{cl} \]

with:

- the factor describing the transition between classic and quantum behavior,

\[ x_{cl} = \left( 1 + e^{5(\frac{\pi T}{\varepsilon_F} - 1)} \right)^{-1} \]

- with $\varepsilon_F = \hbar^2 k_F^2 / 2m_n^*$ the Fermi energy at zero T.

For normal, unbound neutrons:

\[ C_V^q = \frac{1}{6} \left( \frac{2m_n^*}{\hbar^2} \right)^{3/2} \varepsilon_F^{1/2} T \times \left[ 1 - \frac{7}{40} \left( \frac{\pi T}{\varepsilon_F} \right)^2 - \frac{155}{896} \left( \frac{\pi T}{\varepsilon_F} \right)^4 \right]. \]
Thermalization time and specific heat of neutron stars crust

$^{1}S_{0}$ neutron pairing

### Parametrization of $C_{V}^{n}$

$C_{V}^{n} = x_{cl} R C_{V}^{q} + (1 - x_{cl}) C_{V}^{cl}$

with:

- For classic neutrons:

  $$C_{V}^{cl} = \frac{3}{2} \rho_{gas},$$

  with, for $T < T_{gas} = 5.5 \text{ MeV},$

  $$\rho_{gas} = \rho_{n}(T = 0) + \frac{T}{T_{gas}} (\rho_{max} - \rho_{n}(T = 0)),$$

  for $T > T_{gas} = 5.5 \text{ MeV},$

  $$\rho_{gas} = \rho_{max}.$$

  with $\rho_{max}$ for neutrons uniformly distributed in the cell.
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**1S_0 neutron pairing**

### Parametrization of $C_V^n$

$$C_V^n = x_{cl} R C_V^q + (1 - x_{cl}) C_V^{cl}$$

with the factor simulating the reduction due to pairing correlations:

$$R = R_{YL}(u) f_1(T, \Delta_o, a_0, a_1, a_3) (1 - f_2(T, \Delta_o, a_0, a_2, a_3)),$$

where

- $R_{YL}(u)$ is the superfluid reduction factor for uniform neutron matter (Levenfish et al., 1994),
- $f_1$ & $f_2$ are two functions describing the normal/superfluid transition, depending on:
  - $\Delta_o$ the pairing energy gap in the neutron gas at $T=0$,
  - $a_0, a_1, a_2, a_3$ four parameters fitted to reproduce the results the HFB calculations.
Scaling relations

\[ t_w = \alpha t_1 \quad \text{with} \quad \alpha = \left( \frac{\Delta R_{\text{crust}}}{1 \text{ km}} \right)^2 \left( 1 - \frac{2GM}{c^2 R} \right)^{-3/2} \]

\begin{center}

\begin{tabular}{|l|c|c|}
\hline
\textbf{Model of neutron superfluidity} & \textbf{\( t_w \)} & \textbf{\( t_1 \)} \\
\hline
No superfluidity & 76.3 & 66.4 \\
Weak pairing & 43.1 & 37.4 \\
Strong pairing & 24.7 & 21.5 \\
\hline
\end{tabular}

\end{center}

\( M = 1.5M_\odot \) & \( T_i = 5 \times 10^9 \) K