Isospin symmetry breaking in mirror nuclei

Experimental and theoretical methods

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2. Experimental techniques for mirror spectroscopy
Isospin symmetry manifests better along the N=Z line

Analogue states with low spin are studied in CDE (IMME)

What about the difference in excitation energy with increasing spin?

CED have been restricted for many years to low-spin states due to the difficulties in populating proton rich nuclei...

**Experimental issues**

- proton-rich $T_z < 0$ isobars only weakly populated
- "mirrored" gamma-ray energies almost identical
  
  → we need very clean reaction channel selection...
Populating proton-rich nuclei

Fusion-evaporation reactions

\[ \text{target nucleus} \rightarrow \text{compound nucleus} \]

- \( ^{206}_{52} \)Pt \( \rightarrow ^{206}_{51} \)X \( (6^+ \rightarrow 5^+) \)
- \( ^{23}_{9} \)Ne \( \rightarrow ^{23}_{8} \)He \( (9^+ \rightarrow 8^+) \)
- \( ^{30}_{4} \)C \( \rightarrow ^{30}_{3} \)He \( (5^+ \rightarrow 4^+) \)
- \( ^{64}_{7} \)Ge \( \rightarrow ^{64}_{6} \)Ge \( (7^+ \rightarrow 6^+) \)

Gate: \( ^{206}_{51}, ^{647}_{7}, ^{48}_{94} \)

- \( ^{48}_{20} \)Mn \( \rightarrow ^{48}_{19} \)Ca \( (9^+ \rightarrow 8^+) \)
- \( ^{55}_{25} \)Co \( \rightarrow ^{55}_{24} \)Ti \( (6^+ \rightarrow 5^+) \)

- \( ^{55}_{24} \)Ti \( \rightarrow ^{55}_{23} \)V \( (7^+ \rightarrow 6^+) \)
- \( ^{51}_{20} \)Fe \( \rightarrow ^{51}_{19} \)Sc \( (6^+ \rightarrow 5^+) \)

- \( ^{51}_{20} \)Sc \( \rightarrow ^{51}_{19} \)Ca \( (7^+ \rightarrow 6^+) \)
- \( ^{49}_{23} \)Sc \( \rightarrow ^{49}_{22} \)Ti \( (8^+ \rightarrow 7^+) \)

- \( ^{47}_{20} \)Ti \( \rightarrow ^{47}_{19} \)Sc \( (9^+ \rightarrow 8^+) \)

- \( ^{45}_{20} \)Ti \( \rightarrow ^{45}_{19} \)Sc \( (10^+ \rightarrow 9^+) \)

- \( ^{43}_{20} \)Ti \( \rightarrow ^{43}_{19} \)Sc \( (11^+ \rightarrow 10^+) \)

- \( ^{41}_{20} \)Ti \( \rightarrow ^{41}_{19} \)Sc \( (12^+ \rightarrow 11^+) \)

- \( ^{39}_{20} \)Ti \( \rightarrow ^{39}_{19} \)Sc \( (13^+ \rightarrow 12^+) \)

- \( ^{37}_{20} \)Ti \( \rightarrow ^{37}_{19} \)Sc \( (14^+ \rightarrow 13^+) \)

- \( ^{35}_{20} \)Ti \( \rightarrow ^{35}_{19} \)Sc \( (15^+ \rightarrow 14^+) \)

- \( ^{33}_{20} \)Ti \( \rightarrow ^{33}_{19} \)Sc \( (16^+ \rightarrow 15^+) \)

- \( ^{31}_{20} \)Ti \( \rightarrow ^{31}_{19} \)Sc \( (17^+ \rightarrow 16^+) \)

- \( ^{29}_{20} \)Ti \( \rightarrow ^{29}_{19} \)Sc \( (18^+ \rightarrow 17^+) \)

- \( ^{27}_{20} \)Ti \( \rightarrow ^{27}_{19} \)Sc \( (19^+ \rightarrow 18^+) \)

10^{-22} \text{ s} \quad \text{compound formation}

10^{-19} \text{ s} \quad \text{particle evaporation}

10^{-15} \text{ s} \quad \gamma \text{ emission}

10^{-9} \text{ s} \quad \text{ground state}

N=Z

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Example: the \( f_{7/2} \) shell

The \( 1f_{7/2} \) shell is isolated in energy from the rest of fp orbitals.

Wave functions are dominated by \((1f_{7/2})^n\) configurations.

High-spin states experimentally reachable

\[
\begin{array}{c}
\begin{array}{c}
\frac{5}{2} \\
\frac{1}{2} \\
\frac{3}{2}
\end{array} \\
\begin{array}{c}
\frac{3}{2} \\
\frac{1}{2}
\end{array}
\end{array}
\]

Experimental issue: proton-rich \( T_z = -1/2 \) isobars are weakly populated.

“Mirrored” gamma ray energies almost identical – need very clean reaction channel selection...
Experimental requirements

High efficiency and resolution for $\gamma$ detection

Low cross section at high spin (small masses)

High energy transitions

Good selectivity: particle detectors

Many channels opened: high efficient charged-particle detectors

Kinematics reconstruction for Doppler broadening

Mass spectrometers

Neutron detectors to select proton-rich channels

Polarimeters and granularity ($J$, $\pi$, $\delta$)
Gamma spectroscopy

Constructing a level scheme

\[ \gamma_1 \]

\[ \gamma_2 \]

\[ \gamma_3 \]

\[ ^{156}\text{Dy} \]

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Gamma-ray spectrometers

Conventional techniques

New technique: tracking

\[ \varepsilon \sim 10 \rightarrow 5 \% \quad (M_\gamma = 1 \rightarrow M_\gamma = 30) \]

\[ \varepsilon \sim 40 \rightarrow 20 \% \quad (M_\gamma = 1 \rightarrow M_\gamma = 30) \]

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Techniques for proton-rich spectroscopy

Three basic techniques for selecting proton-rich systems

1. High efficiency & high granularity gamma-ray spectrometer
   high fold $\gamma^n$ (n $\geq$ 3) coincidence spectroscopy

2. Gamma-ray array + mass spectrometer + focal plane detectors - identify A,Z of recoiling nucleus and ToF
   $\rightarrow$ tag emitted gamma-rays

3. Identify cleanly all emitted particles from reaction - needs a charged-particle detector array + high-efficiency & high granularity neutron detector array + $\gamma$-ray array
1. High-fold $\gamma$-coincidence spectroscopy

Rely on the power of the array:
- high-fold gamma ray coincidences
- high granularity…
and on the similarity between the energy of the transitions with those of the known mirror nucleus

Double-coincidence spectra after gating on 2 analogue transitions

$^{32}S(^{24}\text{Mg},1p2n)^{53}\text{Co}$

$^{32}S(^{24}\text{Mg},2p1n)^{53}\text{Fe}$

2. Identify A and Z of the recoiling nucleus

- Combined electric and magnetic dipoles → beam rejection & $A/q$ separation
- $A/q$ identified by $x$-position at focal plane
- Z identified by energy loss ($E - \Delta E$) in gas-filled ionisation chamber
- Information used to “tag” coincident gamma-rays at target position
- Efficiency - up to ~ 15%
- Measure the final residue
An example: the A=48 mirror pair

\[ \frac{^{48}Mn_{23}}{^{23}V_{25}} - \frac{^{48}V_{25}}{^{23}Mn_{23}} \]

\[ \sigma\left( ^{48}Mn \right) \sigma\left( ^{48}V \right) \sim 10^{-4} \]

Need very good selectivity

A/q selection at the focal plane + gate on Z=25


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Selecting “pure” spectra

Contaminants can be removed by using the recorded total energy $E$ and time-of-flight (TOF) of the recoils.

Mass is proportional to $ET^2$

$$E = \frac{1}{2}mv^2, \quad v = \frac{d}{TOF}$$

$$\Rightarrow m \propto E(\text{TOF})^2$$

The $ET^2$ information has sufficient resolution to distinguish three mass units difference.

γ-γ coincidence analysis

(A/q = 3, Z=25)-gated and E(ToF)^2-gated

γ-γ coincidence analysis…

M.A. Bentley et al.,

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Fragmentation reactions and exotic beams

Fragmentation reactions with the removal of 5 or more particles are mainly of statistical character and populate yrast states.

One knock-out reactions are a direct process.
Two–proton knockout from neutron-rich nuclei and two–neutron knockout from proton-rich nuclei at intermediate or relativistic bombarding energies are also direct reactions.

Direct reactions selectively populate single-hole states

Between 3 and 5 nucleons removed the two processes compete

Fragmentation reactions are particularly suitable to populate mirror nuclei far from stability and near the proton dripline.
Knockout reactions with exotic beams

**Example:** study the “magicity” of $^{36}$Ca – mirror of magic $^{36}$S (N=20, Z=16)

One neutron removal reaction from $^{37}$Ca beam

Technique pioneered at GANIL (Stanoiu et al. PRC 69, 034312 (2004))
Mirrored fragmentation of N=Z nuclei

MSU experiment

600 mg/cm^2 Be target

S800 Spectrograph

Primary → N=Z second. → “mirrored frag.”

58Ni → 56Ni → 53Mn

Mirror nuclei with multinucleon transfer

A & Z identification

Prisma + CLARA @LNL, Italy

Efficiency ~ 3 %
Peak/Total ~ 50 %
FWHM ~ 10 keV @ v/c = 10%

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Mirror nuclei with multinucleon transfer

$^{32}S + ^{58}Ni \rightarrow ^{29}P + ^{61}Cu$

$^{29}Si + ^{61}Zn$

@ 135 MeV  LNL, N. Marginean

PRISMA

CLARA

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3. Measuring the evaporated particles

With this method we do not measure directly the final residue but the particles emitted from the compound nucleus.

We need detectors with high efficiency.

Advantage: more flexible than recoil mass spectrometry → more channels can be measured!

Disadvantage: not as clean as RMS
If neutrons are needed, it may be much less efficient.
Charged-particle detectors

**DIAMANT**

- 86 CsI(Tl) elements scintillators
- Efficiency: protons ~70%  
  alphas ~ 50%

**EUCLIDES**

- Si E-ΔE telescopes
- Efficiency: protons ~70%  
  alphas ~ 40%
Neutron detection systems

Detectors placed downstream of the target position

Large volume liquid scintillators coupled to photo-multipliers tubes.
Usually replace some of the forward-most Ge detectors of the array

Efficiency ~ 25%

EXOGAM + N-Wall @ GANIL

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Discrimination using time-of-flight data

Problem: one neutron scattered between two detectors looks like two neutrons…

A single scattered neutron $\rightarrow$ different times-of-flight recorded

Genuine 2-neutron event $\rightarrow$ similar time-of-flight recorded
An example: production of $^{50}$Fe

Experiment for $^{50}$Fe, LNL

$$^{28}_{14}\text{Si}_{14} + ^{28}_{14}\text{Si}_{14} \rightarrow ^{56}_{28}\text{Ni}^{*}_{28} \rightleftharpoons ^{50}_{24}\text{Cr}_{26} + \alpha 2p \quad \sigma_{fus} \sim 300\text{mb}$$

$$^{50}_{26}\text{Fe}_{24} + \alpha 2n \quad \sigma_{fus} \sim 0.3\text{mb}$$

**EUROBALL:** High efficiency ($e_{ph} \sim 8\%$) and high granularity (209 crystals) HpGe array.

26 Clover detectors ($\times 4$ crystals) & 15 Clusters ($\times 7$ crystals).

**ISIS:** Charged-particle detector array - 40 Si E-DE telescopes, total efficiency $e_p \sim 70\%$, $e_a \sim 40\%$

**NEUTRON WALL:** 50 detector elements - BC501A Liquid Scintillator. Efficiency (reaction dependent) $e_{1n} \sim 25\%$.


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First observation of excited states in $^{50}\text{Fe}$

$\sigma(\text{Fe})/\sigma(\text{Cr}) \approx 10^{-4}$


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Spectroscopy with exotic stopped beams

Fragmentation of $^{58}$Ni beam
Secondary beam of $^{54}$Ni in the isomeric state $10^+$

States in $^{54}$Ni known up to the $6^+$,


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Gamma and proton decay of $^{54}$Ni

- lifetime
- branching ratio
- MED
- proton decay


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3. Theoretical tools for CED
First part
Basic Shell Model

The hamiltonian (only two-body forces)

\[ H = \sum_{i=1}^{A} \frac{\vec{p}_i^2}{2m} + \frac{1}{2} \sum_{i,j=1}^{A} V_{ij}(\vec{r}) \]

\[ H = \sum_{i=1}^{A} \left( \frac{\vec{p}_i^2}{2m} + U(r_i) \right) + \sum_{i,j=1}^{A} V_{ij}(|r_i - r_j|) - \sum_{i=1}^{A} U(r_i) = H_0 + H_{res} \]

Configuration

\[ \phi = \frac{1}{\sqrt{A!}} \det \begin{pmatrix} \psi_1(r_1) & \psi_1(r_A) \\ \vdots & \vdots \\ \psi_A(r_1) & \psi_A(r_A) \end{pmatrix} \]

spherical mean field

U(r) is a central (1-body) potential

Centrifugal
Coulomb
Nuclear
Configuration mixing

\[ \phi = \frac{1}{\sqrt{A!}} \det \begin{pmatrix} \psi_1(r_1) & \cdots & \psi_1(r_A) \\ \vdots & \ddots & \vdots \\ \psi_A(r_1) & \cdots & \psi_A(r_A) \end{pmatrix} \]

\[ \mathcal{H} = \begin{pmatrix} \langle \phi_1 | H | \phi_1 \rangle & \langle \phi_1 | H | \phi_2 \rangle & \cdots \\ \langle \phi_2 | H | \phi_1 \rangle & \langle \phi_2 | H | \phi_2 \rangle & \cdots \\ \langle \phi_3 | H | \phi_1 \rangle & \cdots & \cdots \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \end{pmatrix} \]

\[ \Psi = \sum_i c_i \phi_i \]

Mixing of configurations due to the residual interaction

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We limit the space to a reduced set of shells. The Hamiltonian becomes an effective Hamiltonian $H_{\text{eff}}$ that accounts for the missing space.

\[
H_{\text{eff}} = H_m + H_M
\]

- monopole
- Multipole

- “unperturbed” energy of the different configurations in which the valence nucleons are distributed
- determines the single particle energies
- dominant role far from stability

- correlations
- mixing of configurations
- coherence
- energy gains
Effect of the correlations

The multipole Hamiltonian $H_M$ is dominated by the pairing and the quadrupole-quadrupole forces.

The fact that some nuclei display collective behaviour depends on the structure of the spherical field near the Fermi surface for both protons and neutrons.
Ingredients for the Shell Model calculations

1) an inert core
2) a valence space
3) an effective interaction that mocks up the general hamiltonian in the restricted basis

The choice is determined by the limits in computing time and memory: large dimension of the matrices to be diagonalised.

Current codes diagonalize matrices of dimension $\sim 10^{10}$
Shell model and collective phenomena

Shell model calculations in the full fp shell give an excellent description of the structure of collective rotations in nuclei of the f_{7/2} shell.


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Mirror energy differences and alignment

probability distribution for the relative distance of two like particles in the $f7/2$ shell

Shifts between the excitation energies of the mirror pair at the back-bend indicate the type of nucleons that are aligning.

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MED are a probe of nuclear structure:
reflect the way the nucleus generates its angular momentum
Nucleon alignment at the backbending


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Define the operator

\[ A_\pi = \left[ \left( a_\pi^+ a_\pi^+ \right)^J=6 \left( a_\pi a_\pi \right)^J=6 \right]^0 \]

“Counts” the number of protons coupled to J=6

Calculate the difference of the expectation value in both mirror as a function of the angular momentum

\[ \Delta A_{\pi,J} = \langle \Phi_J | A_\pi (Z_>) | \Phi_J \rangle - \langle \Phi'_J | A_\pi (Z_<) | \Phi'_J \rangle \]

In \(^{51}\text{Fe}\) \((^{51}\text{Mn})\) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)
In **odd-mass nuclei**, the type of nucleons that aligns first is determined by the **blocking effect** $\rightarrow$ the even fluid will align first.

What about **even-even** rotating nuclei?

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Alignment in even-even rotating nuclei

Renormalization of the Coulomb m.e.? Only a Coulomb effect?

Can shell model do better?

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\[
\Delta A = \langle \Phi_j | A_{\pi} (Z_{>}) | \Phi_j \rangle - \langle \Phi'_j | A_{\pi} (Z_{<}) | \Phi'_j \rangle
\]

calculate for protons in both mirrors:

- \[ \Delta A_{\pi} \]

Counts the number of aligned protons

In \(^{50}\text{Cr} (^{50}\text{Fe})\) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)

Lecture 2
The end