Isospin symmetry breaking in mirror nuclei

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Energy scales

Scales of energy in Nuclear Physics

- **QCD scale**: $1000 \text{ MeV}$
- **pion $\pi^+$**: $\sim 140 \text{ MeV}$

- **pion-mass scale**: $100 \text{ MeV}$
- **deuteron**: $\sim 2 \text{ MeV}$

- **N-binding scale**: $10 \text{ MeV}$
- **collective**: $\sim 1 \text{ MeV}$

- **MED**: $10-100 \text{ keV}$
The Nuclear landscape

- 287 primordial isotopes exist in Nature
- ~6000 nuclei are bound (3000 are known)
A cut view
Nuclear shapes and excitation modes

Spherical

Deformed

collective motion

non-collective motion

E (MeV)

Deformed Nucleus

Near Spherical Nucleus

147 Gd

Magic

(sph. vib.)

Mid-shell
(ellipsoidal)

Magic

(sph. vib.)

Rotation axis

$R_{4/2} < 2$

$R_{4/2} \approx 2.0$

$R_{4/2} \approx 3.33$

$R_{4/2} \approx 2.0$

$R_{4/2} < 2$
Changing deformation along the band

Rotational band

24^+ → Oblate
22^+ → Backbend
20^+ → j_1
18^+ → j_2
16^+ → R → J = R + j_1 + j_2
14^+ → Prolate
12^+ → Band termination
10^+ → all single spins are aligned
8^+ → Deformed nucleus
6^+ → 4^+ → 2^+ → 0^+

E_γ (keV)
J(ℏ)

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The physics at the yrast line

Isospin symmetry
Outline

What’s isospin?
Why isospin symmetry?
How do we study isospin symmetry?
Experimental methods
Theoretical methods
What do we learn from the data?
Background

What’s isospin?
Why isospin symmetry?
Symmetries

Symmetries help to understand Nature

Examination of fundamental symmetries: a key question in Physics

- conservation laws

Conserved quantities imply some underlying symmetry enjoyed by the interactions

In Nuclear Physics symmetries help to understand the behaviour of matter…
Symmetries in nuclear physics

Isospin Symmetry: 1932 Heisenberg SU(2)

Spin-Isospin Symmetry: 1936 Wigner SU(4)

Seniority Pairing: 1943 Racah

Spherical Symmetry: 1949 Mayer

Nuclear Deformed Field (spontaneous symmetry breaking) Restore symm. \( \Rightarrow \) rotational spectra: 1952 Bohr-Mottelson SU(3) Dynamical Symmetry: 1958 Elliott

Interacting Boson Model (IBM dynamical symmetry): 1974 Arima and Iachello

Critical point symm. \( E(5), X(5) \) ….

2000… F. Iachello

K. Heyde
The nucleus: a unique laboratory

Composed by two types of fermions differing only on its charge

Strong interaction: largely independent of the charge

Proton – Neutron exchange symmetry

Proton and neutron can be viewed as two alternative states of the same particle: the nucleon.

The quantum number that characterizes the two charge states is the isospin

\[ t = \frac{1}{2} \]

\[ t_z = -\frac{1}{2} \quad \text{proton} : \pi \]

\[ t_z = +\frac{1}{2} \quad \text{neutron} : \nu \]
1932 Heisenberg applies the Pauli matrices to the new problem of labeling the two alternative charge states of the nucleon.

1937 Wigner: isotopic spin is a good quantum number to characterize isobaric multiplets.

\[ T_z = \sum_{i=1}^{A} t_{z,i} = \frac{N - Z}{2} \]

\[ \left| \frac{N - Z}{2} \right| \leq T \leq \frac{N + Z}{2} \]

• Charge Symmetry: \( V_{pp} = V_{nn} \)
• Charge Independence: \( V_{pp} = V_{nn} = V_{pn} \)
Two-nucleon system

For a two-nucleon system, four different isospin states can exist:

**Triplet** T=1

\[
| T = 1, T_z = 1 \rangle = \uparrow \uparrow \quad \quad | T = 1, T_z = -1 \rangle = \downarrow \downarrow \quad \quad | T = 1, T_z = 0 \rangle = \frac{1}{\sqrt{2}} ( \uparrow \downarrow + \downarrow \uparrow )
\]

**Singlet** T=0

\[
| T = 0, T_z = 0 \rangle = \frac{1}{\sqrt{2}} ( \uparrow \downarrow - \downarrow \uparrow )
\]

Total wavefunction must be antisymmetric on exchange of all (spin, space, isospin) co-ordinates…

\[
\Psi_{\text{space}} \otimes \Psi_{\text{spin}} \otimes \Psi_{\text{isospin}}
\]
Isobaric multiplets

It is easy to extend the formalism to many-nucleon systems.

The isospin quantum number $T$ directly couples together the two effects of charge symmetry/independence and the Pauli principle.

$$T_z = \sum_{i=1}^{A} t_{z,i} = \frac{N - Z}{2} \leq T \leq \frac{N + Z}{2}$$

All nuclear states can be classified by the isospin quantum number, $T$, which to a good approximation can be treated as a good quantum number.

Since $T < T_z$ is forbidden, states of a given $T$ can only occur in a set of nuclei with $T_z = T, T-1, \ldots, -T \rightarrow$ isospin multiplet or “isobaric multiplet”.

This set of states of the same $T$ in such a multiplet are termed “isobaric analogue states” (IAS).
### Energy difference of ground states in A=27

Nuclei of same number of particles A=27

<table>
<thead>
<tr>
<th></th>
<th>27Mg</th>
<th>27Si</th>
<th>27Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.61 MeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.81 MeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>351</td>
<td>351</td>
<td>351</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Protons</th>
<th>Neutrons</th>
<th>pp-pairs</th>
<th>nn-pairs</th>
<th>np-pairs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>15</td>
<td>66</td>
<td>105</td>
<td>180</td>
<td>351</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>14</td>
<td>78</td>
<td>91</td>
<td>182</td>
<td>351</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>13</td>
<td>91</td>
<td>78</td>
<td>182</td>
<td>351</td>
</tr>
</tbody>
</table>

*pn-pairs can exist in states NOT allowed for pp or nn pairs - PAULI PRINCIPLE*

27Mg is less bound than 27Al even though it has less protons, but it has also less p-n T=0 pairs → T=0 stronger than T=1

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R.F. Casten

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Analog states in isospin doublets

Mirror nuclei with $T_z = \pm 1/2$

Test of isospin symmetry

<table>
<thead>
<tr>
<th>$^{27}<em>{13}$Al$</em>{14}$</th>
<th>$^{27}<em>{14}$Si$</em>{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.14 5.67 9$^*$</td>
<td>5.06 5.21 3$^*$</td>
</tr>
<tr>
<td>5.16 5.25 5$^<em>$ 3$^</em>$</td>
<td>4.70 4.5 5$^*$</td>
</tr>
<tr>
<td>4.81 5$^*$</td>
<td>4.29 4.45 11$^<em>$ 5$^</em>$</td>
</tr>
<tr>
<td>4.51 4.58 7$^<em>$ 11$^</em>$</td>
<td>4.14 1$^<em>$ (3$^</em>$)</td>
</tr>
<tr>
<td>4.41 4.05 1$^<em>$ 2$^</em>$</td>
<td>3.80 3$^*$</td>
</tr>
<tr>
<td>3.68 1$^*$</td>
<td>3.54 1$^*$</td>
</tr>
<tr>
<td>2.98 3.09 9$^<em>$ 3$^</em>$</td>
<td>2.87 2.91 9$^<em>$ (3,5)$^</em>$</td>
</tr>
<tr>
<td>2.73 5$^*$</td>
<td>2.65 5$^*$</td>
</tr>
<tr>
<td>2.21 7$^*$</td>
<td>2.16 7$^*$</td>
</tr>
<tr>
<td>0.84 1.01 3$^<em>$ 1$^</em>$</td>
<td>0.78 0.96 3$^<em>$ 1$^</em>$</td>
</tr>
</tbody>
</table>

stable 4.16s
The nucleus can be characterized by isospin quantum numbers which restrict the possible states in which the many-nucleon system can exist.

Look at the isobaric triplet:

\[
\begin{align*}
\frac{12}{12}Mg_{10} & \quad \frac{11}{11}Na_{11} & \quad \frac{10}{10}Ne_{12} \\
\downarrow & \quad \uparrow & \quad \uparrow
\end{align*}
\]

We expect:
- \( T=1 \) states low in energy in \( ^{22}\text{Mg} \) and \( ^{22}\text{Ne} \)
- \( T=0 \) and \( T=1 \) states in \( ^{22}\text{Na} \) (N=Z)

Tests isospin independence
We expect: $T=1$ states low in energy in $^{20}\text{Na}$ and $^{20}\text{F}$

$T=0$ and $T=1$ states in $^{20}\text{Ne}$ (N=Z)

Tests isospin independence
We can form a doublet T=1/2

\[ \left| T = \frac{1}{2}, T_z = +\frac{1}{2} \right\rangle = \uparrow \uparrow \downarrow \downarrow \quad \left| T = \frac{1}{2}, T_z = -\frac{1}{2} \right\rangle = \uparrow \downarrow \downarrow \uparrow \]

or a quadruplet T=3/2

\[ \left| T = \frac{3}{2}, T_z = +\frac{3}{2} \right\rangle = \uparrow \uparrow \uparrow \uparrow \quad \left| T = \frac{3}{2}, T_z = -\frac{3}{2} \right\rangle = \downarrow \downarrow \downarrow \downarrow \]

\[ \left| T = \frac{3}{2}, T_z = +\frac{1}{2} \right\rangle = \uparrow \uparrow \downarrow \downarrow \quad \left| T = \frac{3}{2}, T_z = -\frac{1}{2} \right\rangle = \uparrow \downarrow \downarrow \uparrow \]
T=3/2 Isobaric quadruplets: the spectra

SMALL differences in excitation energy due mainly to Coulomb effects

LARGE differences in mass/binding energy - also due mainly to Coulomb effects....

Courtesy M.A. Bentley
CDE : For any two members of a multiplet of isospin T, transformed through exchange of k protons for neutrons is given by

\[
(CDE)_{\alpha,T,T_z} = M_{\alpha,T,T_z} + k - M_{\alpha,T,T_z} + k\Delta M_{np}
\]

\(\alpha = \) Isobaric analogue state, defined by A, T, J)
A simple relationship between mass and $T_z$ for a set of analogue states (E.P.Wigner 1957)

To derive it we start with eigenstates of a charge-invariant (CI) Hamiltonian. The eigenvalues do not depend on $T_z$ and analogue states are degenerate.

$$H_{CI} |\alpha T T_z \rangle = E_{\alpha T} |\alpha T T_z \rangle$$

A charge-violating interaction lifts this degeneracy:

$$H_{CV} = \sum_{k=0}^{2} H_{CV}^{(k)}$$

Any two-body force can be written in the isospin space in terms of 3 components:

- **Isoscalar:** $H_{CV}^{(0)} = (V_{nn} + V_{pp} + V_{pn})/3$
- **Isovector:** $H_{CV}^{(1)} = V_{pp} - V_{nn}$
- **Isotensor:** $H_{CV}^{(2)} = V_{pp} + V_{nn} - 2V_{pn}$
The total binding energy can be obtained as:

$$BE(\alpha T T_z) = \langle \alpha T T_z | H_{CI} + H_{CV} | \alpha T T_z \rangle$$

The energy splitting in an isobaric multiplet can be obtained as:

$$\Delta BE(\alpha T T_z) = \langle \alpha T T_z | H_{CV} | \alpha T T_z \rangle$$

Let’s apply the Wigner-Eckart theorem:

$$\langle \alpha T T_z | H_{CV} | \alpha T T_z \rangle = \sum_{k=0}^{2} (-1)^{T-T_z} \left( \begin{array}{cc} T & k & T \\ -T_z & 0 & T_z \end{array} \right) \langle \alpha T | H^{(k)}_{CV} | \alpha T \rangle$$

Calling

$$M^{(k)} = \langle \alpha T | H^{(k)}_{CV} | \alpha T \rangle$$

we obtain …
\[
\Delta BE(\alpha TT_z) = \frac{1}{\sqrt{2T+1}} M^{(0)} + \frac{T_z}{\sqrt{T(2T+1)(T+1)}} M^{(1)} + \frac{3T_z^2 - T(T+1)}{\sqrt{(2T-1)T(2T+1)(T+1)(2T+3)}} M^{(2)}
\]

We can now write the total BE into the form (a, b and c independent on T_z):

\[
BE(\alpha TT_z) = a + bT_z + cT_z^2
\]

- **Isoscalar**, \(~100\)’s MeV (includes the contribution of \(V_{\text{Cl}}\))
- **Isovector** \((V_{pp} \neq V_{nn})\) \(~3-15\) MeV (\(~A^{2/3}\))
- **Isotensor** \((V_{pp} + V_{nn} \neq 2V_{np})\) \(~200-300\) keV

The shift of the BE (mass) in an isobaric multiplet depends quadratically on \(T_z\)
Tests of IMME

IMME widely tested through fitting $T=3/2$ quadruplets. Coefficient of cubic term ($dT_z^3$) should be zero.

The major contribution comes from the Coulomb interaction…

**IMME works beautifully!**

Benenson & Kashy RMP 51(1979)527

Courtesy M.A. Bentley

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The Coulomb contribution to IMME

Consider the nucleus a uniform charged sphere, the Coulomb energy is:

\[ E_C = \frac{3e^2 Z(Z - 1)}{5R_C} = \frac{3e^2}{5r_0 A^{\frac{1}{3}}} \left[ \frac{A}{4} (A - 2) + (1 - A)T_z + T_z^2 \right] \]

This crude estimate (dashed-line in the figure) has to be corrected by exchange terms (Pauli Principle), etc.
The Nolen-Schiffer Anomaly

IMME works well, but...reproducing magnitude of coefficients → historical problem

For two adjacent nuclei,

\[ CDE_{\alpha,T,T_z} = M_{\alpha,T,T_z} - M_{\alpha,T,T_z+1} + \Delta M_{np} \]

\[ = -b - c(2T_z + 1) + \Delta M_{np} \]

Nolen & Schiffer (1969) calculated CDE for wide range of isobaric multiplets...

Used independent-particle models (with exchange term) → Corrected for other phenomena (e.g. electromagnetic spin-orbit (coming soon), magnetic effects, core-polarisation, isospin impurity, etc.).

But...magnitude of predicted CDE ALWAYS ~5-8% lower than experimental values. Underestimated by ~500 keV!

Auerbach (1983) improves the theoretical description but the anomaly remained

Duflo and Zuker (2002) reduce the difference to ~100-200 keV
What about excited states - **Coulomb Energy Differences**…

“Normalise” ground state energies and take differences in excitation energy (CED)

How does Coulomb energy (+ other INC) change with $E_x$, J ?

Does Nolen-Schiffer Anomaly only concern ground states?
CED: MED and TED

Mirror Energy Differences

\[ \text{MED}_J = E_{x J, T_z = -1/2} - E_{x J, T_z = +1/2} = -\Delta b_J \]

Test the charge symmetry of the interaction

Triplet Energy Differences

\[ \text{TED}_J = E_{x J, T_z = -1} + E_{x J, T_z = +1} - 2E_{x J, T_z = 0} = -2\Delta c_J \]

Test the charge independency of the interaction
A classical example: MED in $T=1/2$ states

Coulomb effects in isobaric multiplets:
- bulk energy (100’s of MeV)
- displacement energy (g.s.) CDE (10’s of MeV)
- differences between excited states (10’s of keV)

$\text{Mirror Energy Differences} \quad MED_J = E_J(Z>N) - E_J(Z<N)$
Isospin symmetry breakdown, mainly due to the Coulomb field, manifests when comparing mirror nuclei. This constitutes an efficient observatory for a direct insight into nuclear structure properties.
We will see that CED are extremely sensitive to quite subtle nuclear structure phenomena which, with the aid of shell model calculations, can be interpreted quantitatively at the level of 10’s of keV.

We measure *nuclear* structure features:

- How the nucleus generates its angular momentum
- Evolution of the radii (deformation) along a rotational band
- Learn about the configuration of the states
- Isospin non-conserving terms in the nuclear interaction
Understanding structure features

Most of the structure features of nuclei in the f_{7/2} shell are very well described by shell model calculations in the full fp valence space.

What happens at the backbending?
- band-crossing?
- alignment?
- which nucleons are aligning?

S.M. Lenzi et al., *PRC 56*, 1313 (1997)
Shifts between the excitation energies of the mirror pair at the back-bend indicate the type of nucleons that are aligning.


Probability distribution for the relative distance of two like particles in the $f7/2$ shell.
Nucleon alignment at the backbending

**Experimental MED**

MED are a probe of nuclear structure: reflect the way the nucleus generates its angular momentum.

Nucleon alignment at the backbending


Alignment

MED (keV)

Experiment
Shell Model

Energy (MeV)

51Fe

51Mn

MED

Alignment

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Cranked shell model and alignment

Rotational dependence of Coulomb energy differences

J.A. Sheikh, D.D. Warner and P. Van Isacker

Cranked shell-model

\[ H' = h_{def} + V_2 - \omega J_x \]

Approximations:
- one shell only
- fixed deformation
- no p-n pairing

CSM: good qualitative description of the data

J.A. Sheikh, P. Van Isacker, D.D. Warner and J.A. Cameron,

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Define the operator
\[
A_\pi = \left[ \left( a^+_\pi a^+_\pi \right)^{J=6} \left( a^+_\pi a^+_\pi \right)^{J=6} \right]^0
\]

“Counts” the number of protons coupled to J=6

Calculate the difference of the expectation value in both mirror as a function of the angular momentum

\[
\Delta A_{\pi,J} = \langle \Phi_J | A_\pi (Z_\pi) | \Phi_J \rangle - \langle \Phi_J | A_\pi (Z_\pi) | \Phi'_J \rangle
\]

In $^{51}$Fe ($^{51}$Mn) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)
Alignment in odd- and even-mass nuclei

In odd-mass nuclei, the type of nucleons that aligns first is determined by the blocking effect → the even fluid will align first.
Alignment in even-even rotating nuclei

In $^{50}$Cr ($^{50}$Fe) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons).

Renormalization of the Coulomb m.e.? Only a Coulomb effect?

Can shell model do better?


Lecture 1
The end